How Reading Research Can Inform Mathematics Difficulties:

The Search for the Core Deficit

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Gersten, Jordan, and Flojo's (in this issue) article is important and provocative in several ways. First, they have done an excellent job in discussing some of the characteristics of young children with mathematics difficulties (MD). Second, they have presented some practical implications of their findings. More specifically, they have identified some promising measures for the early identification of children at risk for MD, as well as instructional strategies that show potential as approaches for intervention. These are important contributions, and they point the field in promising directions for both understanding and meeting the needs of children with MD. However, they acknowledge that the study of MD is in its infancy, whereas reading research has matured and shown tremendous progress in the last 25 years. In this commentary, I will discuss how the assumptions, models, and methodologies developed by reading researchers can guide MD research. By heeding lessons learned from the field of reading disabilities (RD), investigators can avoid many of the challenges that reading researchers had to overcome.

Gersten et al. (in this issue) have outlined the wide array of deficits shown by children with MD. This includes low mastery of and fluency in the retrieval of arithmetic combinations, slow digit naming speeds, inefficient and immature counting strategies, low number sense, and impaired nonverbal working memory. Reading researchers were also faced with a broad range of tasks that differentiated children with RD. For example, children with RD showed impaired performance on tasks that assessed phonological processing, vocabulary, syntactic processing, and verbal working memory. However, reading researchers took the important step of distinguishing between those factors that play a causal role in reading failure and those that are consequences of these factors. There is considerable consensus that the causal deficit in RD is impaired phonological processing, which is most evident as impaired phonological awareness—the ability to identify and manipulate phonemes (Adams, 1990; Cunningham & Stanovich, 1997; Stanovich & Siegel, 1994). Deficits in phonological awareness directly interfere with the acquisition and mastery of the spelling–sound correspondences that underlie fluent reading (Goswami & Bryant, 1990; Tunmer & Hoover, 1992). However, because children with RD tend to avoid reading, their exposure to print is limited. This provides them with fewer opportunities to acquire the breadth of vocabulary and syntactic structures that skilled readers acquire (Stanovich, 1986). In short, deficits in phonological awareness have a direct relationship with decoding ability and an indirect relationship with other correlates of RD.

By exploiting the variables that play a direct causal role in reading failure, researchers have been able to develop reading readiness tests that have predictive validities in the range of .6 to .7, and an array of interventions that prevent reading difficulties before the onset of formal reading instruction, thereby avoiding the cognitive and linguistic consequences associated with reading failure. The field of MD will make similar progress once investigators can distinguish between those variables that cause MD and those that are consequences of MD. Because mathematics is a broad domain with many subdisciplines, more than one variable may play a causal role in MD. However, it may be possible to identify one that is relevant to the development of arithmetic skills.

The task for the field of MD is therefore to identify a core deficit or deficits. Furthermore, this deficit must satisfy the assumption of specificity (Stanovich, 1993; Stanovich & Siegel, 1994). This is the assumption that children with learning disabilities have a cognitive deficit that involves a domain-specific process, rather than a domain-general one, such as processing speed, general auditory processing, or automaticity. The concept of learning disabilities requires that the deficits not extend too far into other domains of cognitive abilities. Otherwise, the deficits would depress cognitive functioning in all domains, and not just, say, reading or mathematics. In short, if the cause of MD fails to satisfy the assumption of specificity, significant difficulties in mathematics would be found only in individuals with low intelligence, which research suggests is not the case (Siegel, 1988; Stanovich, 1999). The underlying deficit of MD must therefore be assumed to be a modular process rather than a domain-general process with widely distributed effects (see Coltheart, 1999; Fodor, 1983).

One potential candidate for the core deficit underlying MD, particularly for children with deficits in arithmetic, may be the representation of number (Ansari & Karmiloff-Smith, 2002). Butterworth (1999) has proposed a number module dedicated to processing quantity. This module is thought to detect changes in quantity and to order quantities by magnitude. Support for the existence of such a module comes from neuropsychological research, longitudinal research, and empirical investigations of conceptual development during infancy—the same sources used to identify the existence of a phonological module.

To begin with, neurophysiological research has provided evidence for the existence of a number module. For example, both functional neuroimaging studies and investigations of individuals who have sustained localized brain damage suggest that specific brain regions are dedicated to processing numerical stimuli (Dehaene, 1996; Dehaene & Cohen, 1995; Grafman, Passafiume, Faglioni, & Boller, 1982; McCloskey, Harley, & Sokol, 1991). More specifically, a bilateral inferior parietal neural pathway is thought to underlie the manipulation of numerical quantities, whereas a left, subcortical network supports the storage and retrieval of arithmetic combinations (Dehaene & Cohen, 1995). These findings suggest that number processing is both domain specific and neurally localized—features that characterize modular processes.

Another source of support for the view that a module in processing number may be the core deficit underlying MD comes from longitudinal research. Longitudinal studies have been particularly productive in reading research, as there has been considerable convergence in the finding that a deficit in phonological awareness is one of the most robust predictors of subsequent reading skill (Cunningham & Stanovich, 1997; Share, Jorm, Maclean, & Matthews, 2002). Initial findings suggest that the same may hold with MD—screening tests that tap children's skill at representing and manipulating number seem to be robust predictors of subsequent mathematical skills (Gersten et al., in this issue). Indeed, the tests that show the most robust relationships with subsequent performance on standardized tests of mathematics are tests that assess children's understanding of magnitude, counting, and differences in quantities. For example, the *Number Knowledge Test* (Okamoto & Case, 1996), which assesses children's understanding of magnitude, discrimination between quantities, and counting, has been found to have predictive validities that range from .64 to .73. Other screening tools that assess these same skills also have predictive validities for performance on standardized mathematics tests 1 year later that range from .50 to .79 (Chard et al., cited in Gersten et al., in this issue; Clarke & Shinn, 2004). The predictive validity of these measures is comparable to the predictive validity of measures of phonological awareness. Thus, longitudinal studies with school-age children support the view that deficits in processing number may play a causal role in MD.

In addition to neurological and behavioral studies of mathematics in school-age children and adults, researchers have examined the development of the representation and processing of numerical stimuli across infancy and early childhood. Like the rapid changes found in infants' phonological representations (e.g., Werker & Tees, 1984), young infants appear to have primitive representations of number that change rapidly over the first year of life (Ansari & Karmiloff-Smith, 2002). A number of experiments has found that neonates can represent quantities of 3 or fewer (Antell & Keating, 1983; Strauss & Curtis, 1981). At 4 months, infants are sensitive to changes in quantity or number operations involving quantities of 3 or fewer (Wynn, 1992). At 11 months, infants can discriminate between larger numbers that are separated by large quantities, such as sets of 8 and 16 stimuli, but not similar quantities, such as sets of 8 and 12 stimuli (Carey, 2001). These precursor representations provide a foundation for the development of mature representations of number. Delays or failures in the acquisition of mature representations may underlie the range of deficits shown by children with MD, just as interruptions in the development of mature phonological representations contribute to the range of deficits that characterize RD.

Ansari and Karmiloff-Smith (2002) have argued that two systems may be involved in the representation of number: an *analog* system that approximates real quantity and functions independently of language and culture, and an *exact* system that has precise representations and is dependent on language and culture. The analog system is thought to support infants' initial representations, although there is currently debate about the nature of these numerical representations (Carey, 2001; Gallistel & Gelman, 2000). Despite disagreements about whether magnitudes are represented by unique, nonnumerical mental symbols known as object files or by noisy mental magnitudes with scalar variability (Carey, 2001; Gallistel & Gelman, 2000), there is consensus that the analog system does not accurately represent numerosities greater than 3. In contrast, the exact system is precise and represents numbers in natural languages (Dehaene & Cohen, 1995). The ability to represent numbers greater than 4, understand relative quantities, and know how numbers work would depend on the development of the latter system, in which exact numbers are represented.

The failure to develop mature representations of number could interfere with or delay basic understanding of number. Children who do not represent number using the exact system when instruction in mathematics begins face considerable challenges in developing basic mathematical concepts. For example, because these children have an impoverished understanding of number, they are both inaccurate and unsophisticated in counting (e.g., Geary, 2003; Siegler & Shrager, 1984). Immature representations of number can account for the limited number sense and difficulties in discriminating between quantities of children with MD. For example, Gersten et al. (in this issue) noted that although some children can accurately count up to 5 through rote memorization, they cannot tell whether 4 is larger than 2 or the other way around. Although these children have learned a linguistic sequence (the numbers of 1 to 5 in the correct order), they have failed to represent the number terms as quantities. Therefore, delays or impairments in the development of mature representations of number may interfere with or impede the achievement of number sense.

Failure to develop mature, integer list representations of number can also have repercussions for domain-general processes, which I have called consequences of MD. Although children with MD tend to show impaired performance in nonverbal working memory (Siegel & Ryan, 1988) and slow speeds of processing (Geary & Brown, 1991), their performance on these tasks may be dependent on the quality of their numerical representations. Indeed, the deficits of children with MD in these domains are most evident when the tasks involve numerical information, such as the Digit Span Backward or rapid automatized naming using digits as stimuli. Children whose numerical representations are of low quality or imprecise will experience significant difficulties in encoding, storing, and retrieving numerical stimuli from memory. Thus, immature or analog representations of number would have repercussions for all tasks that use numbers, even if the tasks are meant to tap working memory or processing speed.

In short, the literature suggests that a domain-specific module that represents number may be the cause of MD. I would now like to propose two research paradigms borrowed from reading research that could serve to strengthen the case. The first involves matching children of different ages based on their mathematics ability, which would allow researchers to identify which cognitive deficits are central to MD. The second seeks to identify the core deficit by examining the consequences of in-depth instruction on number representation to see if ameliorating deficient representations leads to growth in mathematics. These paradigms will be discussed in turn.

A mathematics-level–match design represents an attempt to isolate cognitive deficits that have a direct relationship with MD from those that are consequences of MD. This design parallels the reading-level–match design, which was successful in determining that deficient phonological processing characterizes RD, whereas deficient vocabulary or syntactic processing appear to be consequences of children's reading histories (Bryant & Goswami, 1986; Jackson & Butterfield, 1989). Using the mathematics-level–match design, older children with MD would be compared to younger, typically achieving children who are matched by their performance on standardized tests of mathematics. In other words, although MD and control children's mathematics performance would be matched, they would differ in age. When older children with MD perform lower than younger, mathematics-level controls on a task, group differences cannot be considered a consequence of proficiency in mathematics. Instead, these differences would be thought to reveal deficits in cognitive processing that are directly related to MD. Differences in the patterns of impairment that can be revealed by the chronological age–match and mathematics-level–match designs are illustrated by Keeler and Swanson (2001). They compared children with MD to both chronological age–matched children and younger, mathematics-level–matched children. Although children with MD showed deficits in nonverbal working memory when compared to the chronological age–matched controls, they performed as well as young mathematics-level–matched controls. These findings suggest that the developmental lag of children with MD in nonverbal working memory is not a cause of MD, but rather a consequence of it. Further research using the mathematics-level–match design should be used to examine the nature of the relationships between mathematics achievement and other correlates of MD, such as number sense. For example, support for the diagnosticity of deficient number sense in MD would take the form of older children with MD showing impaired performance relative to younger, mathematics-level–matched controls on tasks such as quantity discrimination or identifying missing numbers from a sequence.

An important caveat with the mathematics-level–match design is that it cannot be used in isolation to infer causation, as older children with MD have very different experiences than younger children who perform at the same level. Instead, it reveals if high- and low-achieving students reach the same level of mathematical proficiency using the same cognitive processes. To establish the causal role of differences revealed by this design, mathematics-level–match studies must be corroborated with longitudinal, intervention, and training studies.

The logic behind intervention and training studies is straightforward. If the deficit that plays a causal role in academic failure is remediated, then improvements should be seen in the relevant academic domain. For example, the moderate to strong effects (*d*s = .53 to .70) of phonological awareness training on reading skills for prereaders, children with RD as well as typically achieving children (Bus & van IJzendoorn, 1999; Ehri et al., 2001), support the hypothesis that impaired phonological awareness plays a causal role in RD. Similarly, researchers may be able to establish if number representation plays a causal role in MD through training studies. If the representation of number is at the core of

MD, then interventions targeting number sense may be critical for improvement in mathematics performance. Gersten et al. (in this issue) have noted that there is a paucity of research on early interventions to prevent MD in at-risk children. This is exactly the type of research we need.

In conclusion, I am very appreciative of Gersten et al.'s (in this issue) thoughtful and thorough presentation of the correlates of MD and their implications for early identification and intervention. However, as I have argued in this commentary, models and methods from reading research ought to be adopted by researchers who wish to answer both theoretical and practical questions about MD. At the heart of this is the need to identify the core deficit underlying MD, which will further the field in both directions.

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